Expected values

Linearity of expected values: E[x] + E[y] = E[x + y], regardless of dependence

Example 1:

choose 10 cards at random from a deck, find expected value of number of aces

Example 2:

Price of TV is $1000, each day go up by $5 or $10 with probability 50% for the next n days, find expected value of final price.

Example 3:

Find the expected value of maximum value of results of 2 dice rolls.

Example 4:

Find the expected value of maximum value of results of n dice rolls.

Solution 1: use previous solution

Solution 2: dynamic programming

dp[n][6], dp[i][j] = i rolls to get maximum of j

If n is huge, can just output 6

Example 5:

There is a class of 20 students with lessons every day in a year. When someone has birthday, the class will be cancelled. What is the expected value of number of cancelled class?

Example 6:

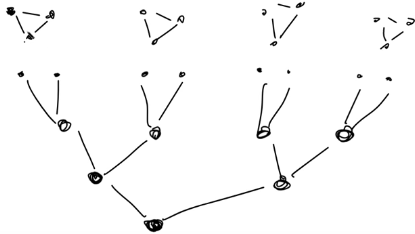
Toss a coin until you get heads. What is the expected value of tosses?

Example 7:

Toss a coin until you get a total of 2 heads. What is the expected value of tosses?

Bonus: toss a coin until you get 2 heads in a row. What is the expected value of tosses?

Example 8:

There are 12 volleyball teams and they are divided into 4 groups of 3. In each group, 2 teams will get to the next stage. Then, the 8 teams will have a tournament. Each team wins with 50% probability. If there is a perfect draw in the groups of 3, 2 random teams will get into the next stage

Contribution technique

If we want to find the sum over many possibilities, we should consider every element and count how many times it will be added or contribute to the answer.

Problem 1: Hills

Given a sequence of length N, count triples of indices i < j < k such that a[i] < a[j] > a[k]

Iterate over j, find number of elements smaller than j before j and number of elements larger than j after j, sum the product of the 2 numbers.

O(N log N) with segment tree

Problem 2: Paths in tree

Given a tree of size N, find the sum of lengths of all paths.

For every edge, add the number of paths containing it to the answer. This works because for any path of length L, the L edges in it will all count.

Use dfs to find subtree size of nodes first

O(N)

Example 3: Math encoder

Given a sequence of N numbers, choose one non-empty subset at random. Find the expected value of the difference between the maximum and the minimum element in the subset.

Sort the sequence, it doesn’t matter because the problem is asking about subsets

Iterate over starting position and ending position of subset, starting position will be the minimum and the ending position will be the maximum as the array is sorted.

Each have 2^(j-i-1) subsets, so add 2^(j-i-1)\*(a[j]-a[i]) each time

BIG BRAIN: Expected value of max – min = Expected value of max – Expected value of min

O(N log N + N)

Example 4: Randomizer

Given a convex polygon with N vertices, choose a random subset of vertices. Find the expected value of the perimeter of the new convex polygon.

If N is large, can only consider edges with length up to K, if is too small.

If ask about area, can use Pick’s theorem, which states that

Example 5: random convex hull

Given N points such that no 3 points are collinear, and each point disappears with a probability of 50%. Find expected value of the size of the convex hull of remaining points.